

- | | |
|---|---|
| (i) It is determinate. | (i) It is indeterminate. |
| (ii) At the supports, slopes θ_A , θ_B are produced. | (ii) At the supports, slopes are zero.
$\theta_A = 0$, $\theta_B = 0$. |
| (iii) At the supports, moments are zero. $M_A = 0$, $M_B = 0$ | (iii) At the supports fixed end moments M_A , M_B are developed. |
| (iv) Deflection at the middle of beam is more. | (iv) Deflection at the middle of beam is less. |
| (v) B. M. at the center of the beam is more than that in case of fixed beam. | (v) B. M. at the center of the beam is less than that in case of simply supported beam. |
| (vi) Strength of simply supported beam is less than the strength of fixed beam. | (vi) Strength of fixed beam is more than the strength of simply supported beam. |

1.3 ADVANTAGES AND DISADVANTAGES OF FIXED BEAM :

(March - 2008, Dec-2011)

A fixed beam has the following advantages over a simply supported beam

1. The beam is stiffer, stronger and more stable.
2. The slope at both ends is zero.
3. The fixing moments are developed at the two ends, whose effect is to reduce the maximum bending moment at the center of the beam.
4. The deflection at the center is very much reduced.

❖ Disadvantages :

1. Both the ends of beam must be at the same level. If there is difference level of two supports, extra stress is produced in the beam.
2. Stresses are produced in the beam due to change in temperature.
3. The fixity of beam is reduced due to vibrations.

Now,

Area of μ - dia. + Area of μ' - dia. = 0

$$\therefore \left(\frac{1}{2} \times 5 \times 50 \right) + M_A \times 5 = 0$$

$$\therefore M_A \times 5 = -125$$

$$\therefore \boxed{M_A = -25 \text{ kN.m}}$$

$$\therefore \boxed{M_B = -25 \text{ kN.m}}$$

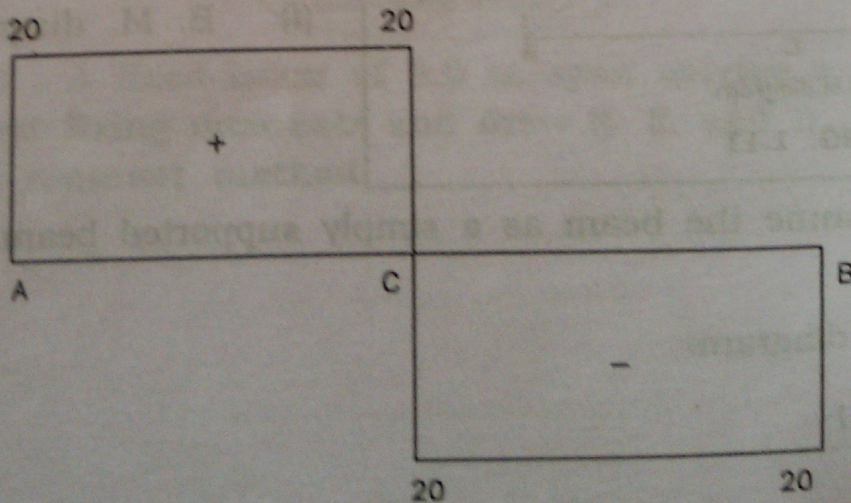
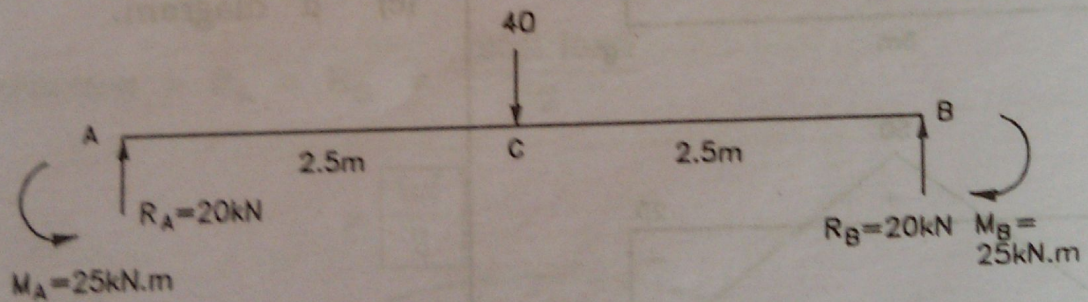
For S. F. diagram :

Each reaction = $R_A = R_B$

$$= \frac{\text{total load}}{2}$$

$$= \frac{40}{2}$$

$$= 20 \text{ kN}$$



S.F. DIAGRAM

FIG. 1.12(A)

Contraflexure (Points E and F) :

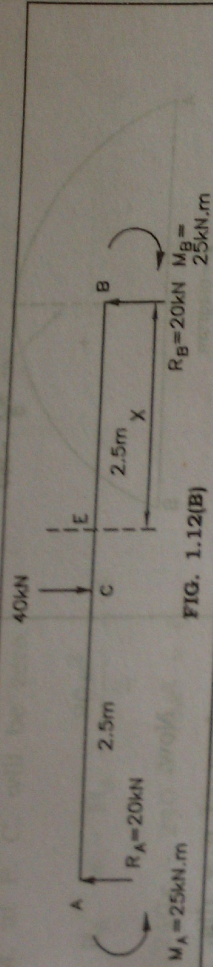


FIG. 1.12(B)

Let P. C. occur at distance x from B.

B. M. at P. C. will be zero.

$$M_E = 20x - M_B = 0$$

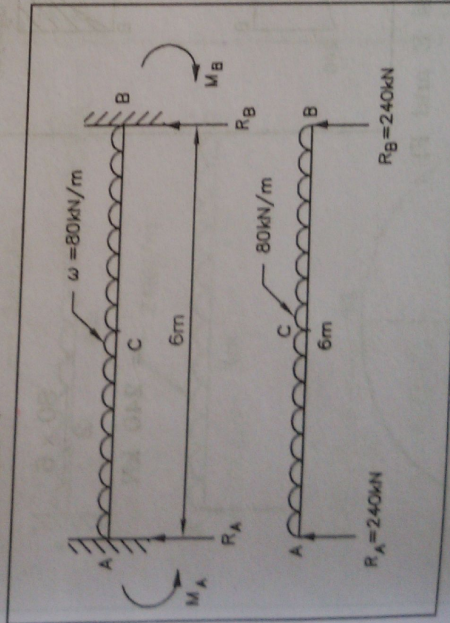
$$\therefore 20x - 25 = 0$$

$$\therefore 20x = 25$$

$$\therefore x = 1.25 \text{ m} \dots \dots \text{ from B or from A}$$

Example - 2 : A fixed beam of 6 m span carries u.d.l. of 80 kN/m over its entire span. Draw S. F. & B. M. diagrams for the beam. Also find point of contraflexure. (Nov.-2006) (Similar June-2013)

Solution :



First of all assume the beam as a simply supported beam and draw B. M. diagram.

i.e. draw μ - diagram.

$$M_C = \frac{\omega l^2}{8} = \frac{80 \times 6^2}{8} = 360 \text{ kN.m}$$

Now, draw B. M. diagram for a beam with only end moments M_A and M_B . This B. M. diagram is called μ' - diagram.

Examples of statically determinate beams are :

- Simply supported beam
- Cantilever beam
- Overhanging beam

❖ **Statically indeterminate beam :** (April – 2006, May – 2007, Dec. – 2008,

A beam which cannot be analysed by using three conditions of equilibrium ($\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$), is known as **statically indeterminate beam**.

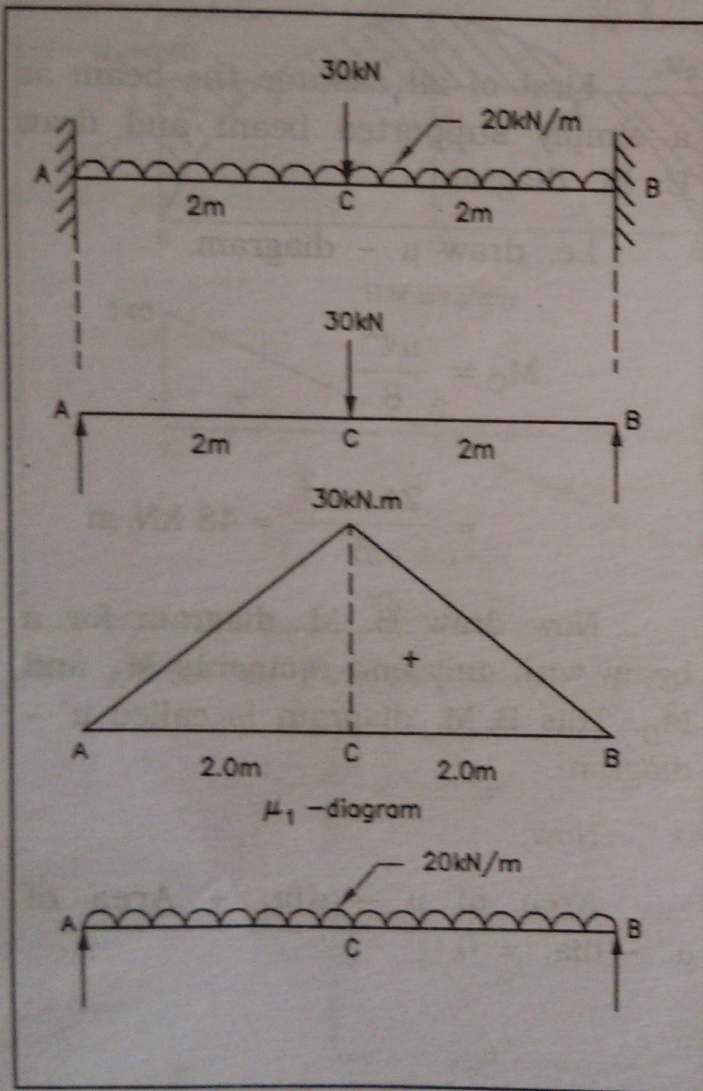
In statically indeterminate beam the number of unknowns are more than the number of equilibrium equations available. i.e. Number of unknowns are normally more than three.

Examples of statically indeterminate beams are :

- Fixed beam
- Continuous beam
- Propped cantilever beam

✓ Example - 4 : A fixed beam of span 4 m carries u.d.l. of 20 kN/m over entire span and a central point load of 30 kN. Draw B.M. diagram for the beam.
 (May - 2007, Similar July-2010, June-2011)

Solution :



First of all assume the beam as a simply supported beam and draw B.M. diagrams, for point load and u.d.l. separately.

μ_1 = B. M. dia. for point load

μ_2 = B. M. dia. for u.d.l.

$$(A) \quad M_C = \frac{Wl}{4} = \frac{30 \times 4}{4} = 30 \text{ kN.m}$$

$$(B) \quad M_C = \frac{wl^2}{8} = \frac{20 \times 4^2}{8} = 40 \text{ kN.m}$$

Now draw μ' - diagram.

Area of μ_1 - dia. + Area of μ_2 - dia.
 + Area of μ' - dia. = 0

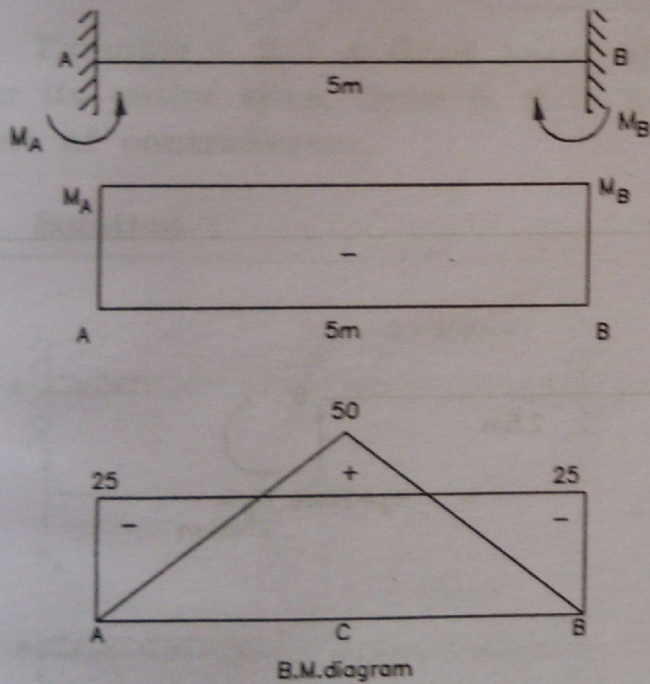
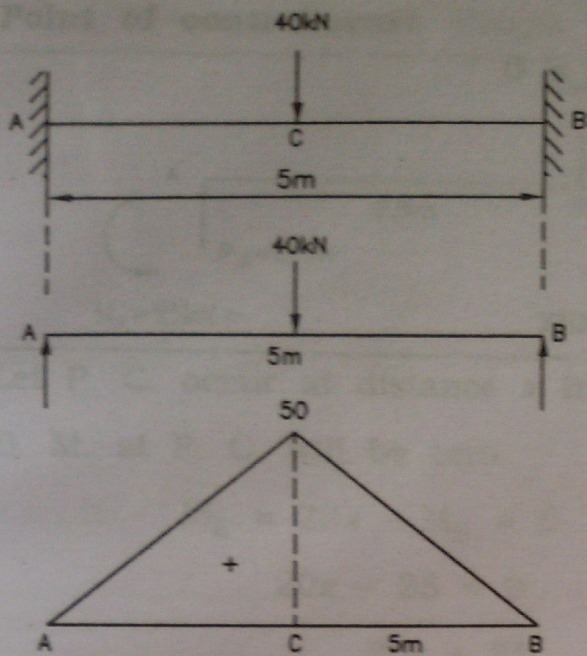


FIG. 1.11

(a) Load diagram.

(b) Simply Supported beam with central point load.

$$M = \frac{Wl}{4} = \frac{40 \times 5}{4} = 50 \text{ kN.m}$$

(c) μ diagram.

(d) Fixed beam with fixing moments.

(e) μ' diagram.

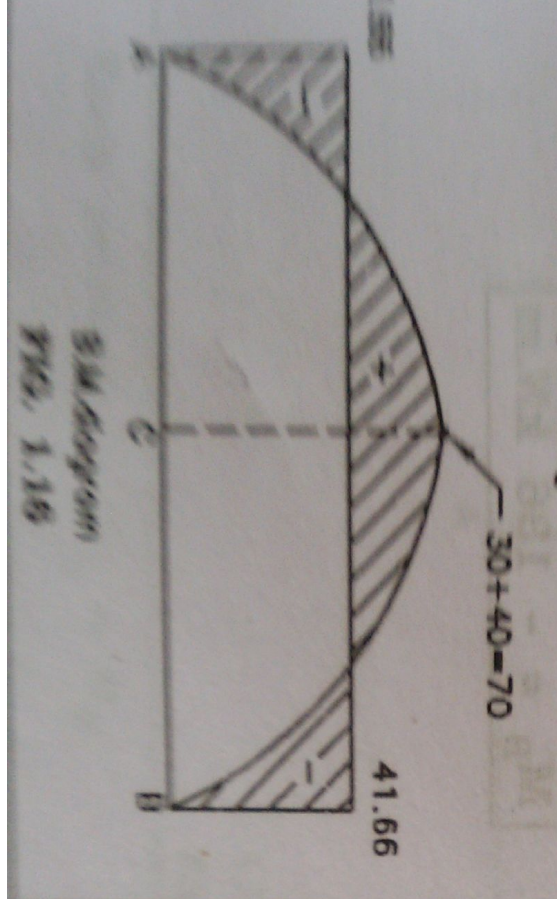
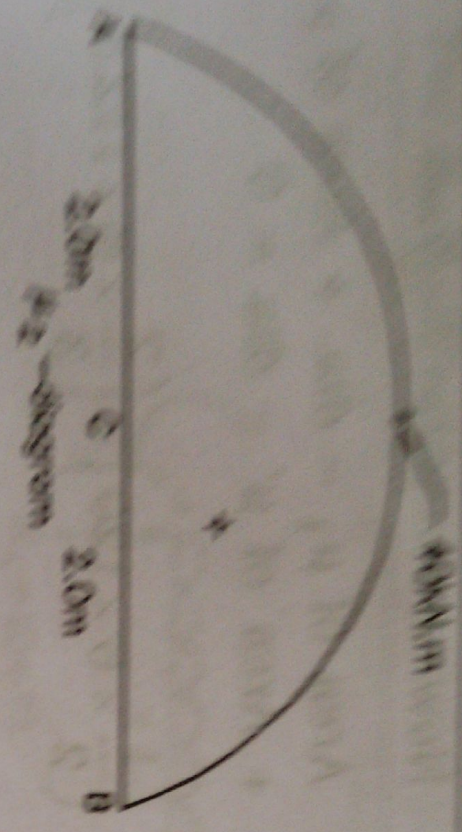
(f) B. M. diagram.

First of all assume the beam as a simply supported beam and draw B. M. dia.

i.e. draw μ - diagram.

$$M_C = \frac{Wl}{4}$$

$$= \frac{40 \times 5}{4} = 50 \text{ kN.m}$$



$$\therefore \left(\frac{1}{2} \times 4 \times 30 \right) + \left(\frac{2}{3} \times 4 \times 40 \right) + M_A \times 4 = 0$$

$$\therefore 60 + 106.66 = - M_A \times 4$$

$$\therefore M_A = - 41.66 \text{ kN.m}$$

$$\therefore M_B = - 41.66 \text{ kN.m}$$

Example - 6 : A fixed beam AB of span 6 m carries two point loads of 20 kN each at distance 1.5 m from each end. Draw S. F. and B. M. diagram for the beam. (Oct. - 2000)

Solution :

First of all assume the beam as a simply supported beam and draw B. M. dia.

i.e. draw μ - diagram.

Each reaction for simply supported beam,

$$R_A = R_B = \frac{(20 + 20)}{2} = 20 \text{ kN}$$

$$\therefore M_C = 20 \times 1.5 = 30 \text{ kN.m}$$

$$M_D = 20 \times 1.5 = 30 \text{ kN.m}$$

Now draw μ' - dia. For a fixed beam with only end moments M_A and M_B .

Area of μ - dia. + Area of μ' - dia. = 0

$$\therefore 2 \times \left[\frac{1}{2} \times 1.5 \times 30 \right]$$

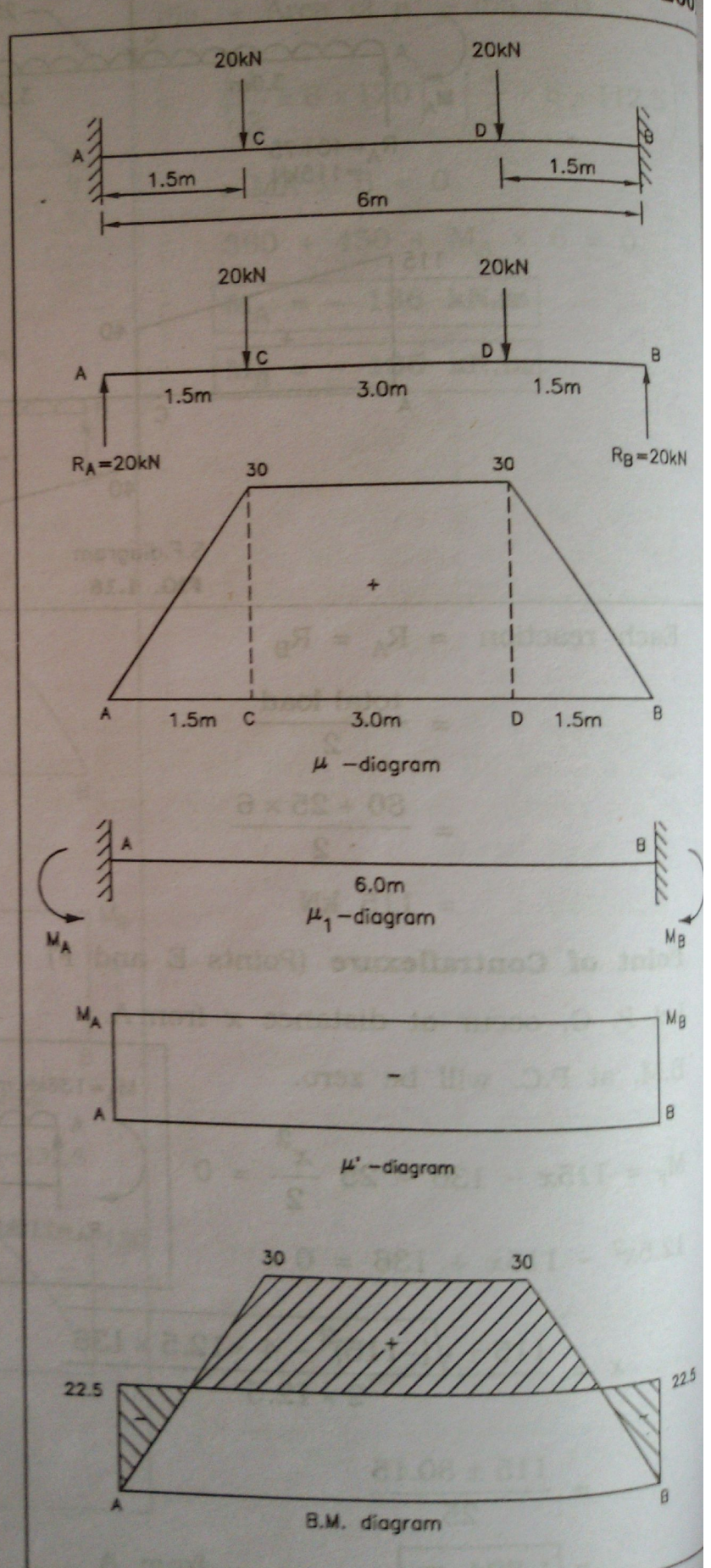
$$+ 30 \times 3 + M_A \times 6 = 0$$

$$\therefore 45 + 90 + M_A \times 6 = 0$$

$$\therefore M_A \times 6 = -135$$

$$\therefore \boxed{M_A = -22.5 \text{ kN.m}}$$

$$\therefore \boxed{M_B = -22.5 \text{ kN.m}}$$



Let P. C. is at distance x from A.
B. M. at P. C. will be zero.

$$\therefore R_A \cdot x - M_A - \frac{80x^2}{2} = 0$$

$$\therefore 240x - 240 - 40x^2 = 0$$

$$\therefore x^2 - 6x + 6 = 0$$

$$\therefore x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$= \frac{6 \pm 3.46}{2}$$

$$= \boxed{1.27 \text{ m}}$$

or

$$\boxed{4.73 \text{ m}}$$

... .. from A

1.2 DISTINGUISH BETWEEN FIXED BEAM AND SIMPLY SUPPORTED BEAM :

(July-2010, Dec.-2011, Dec.-2012, June-2013)

Simply Supported Beam

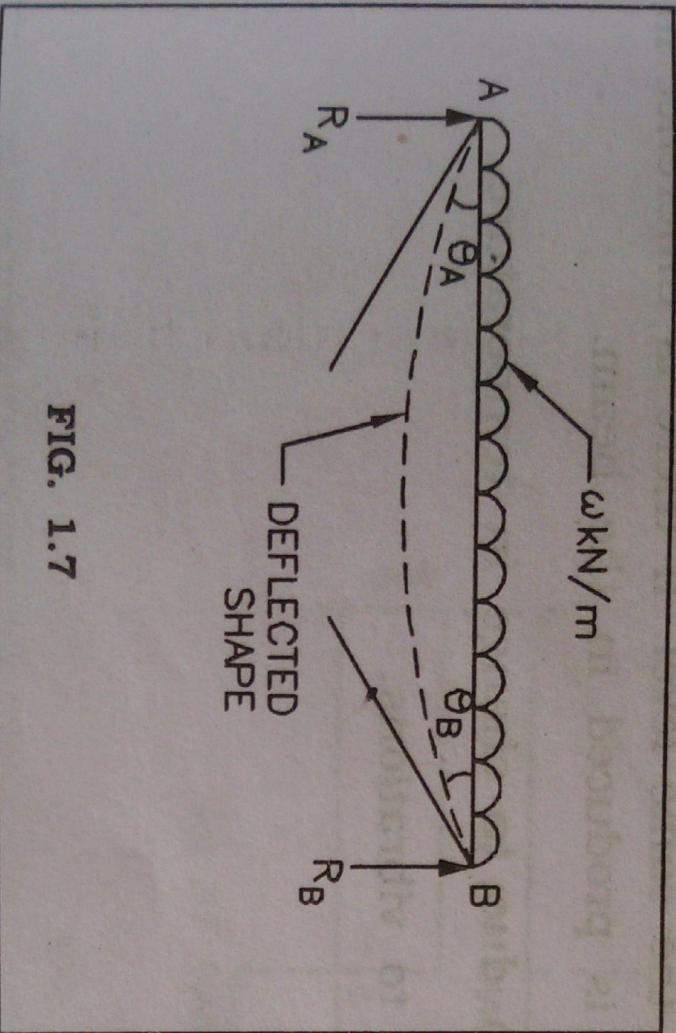


FIG. 1.7

Fixed Beam

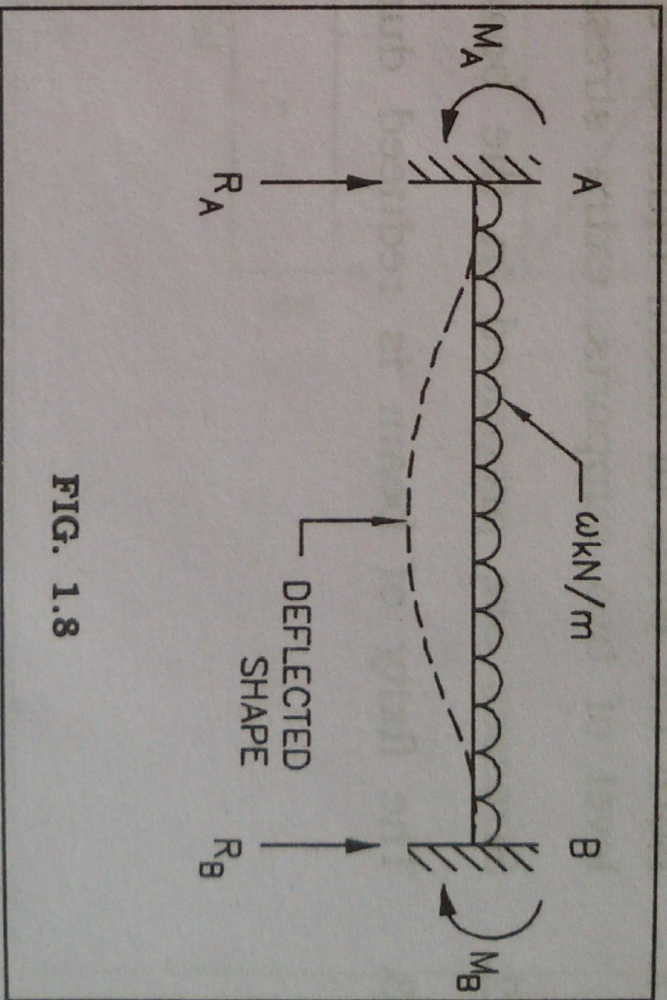


FIG. 1.8

S. F. diagram :

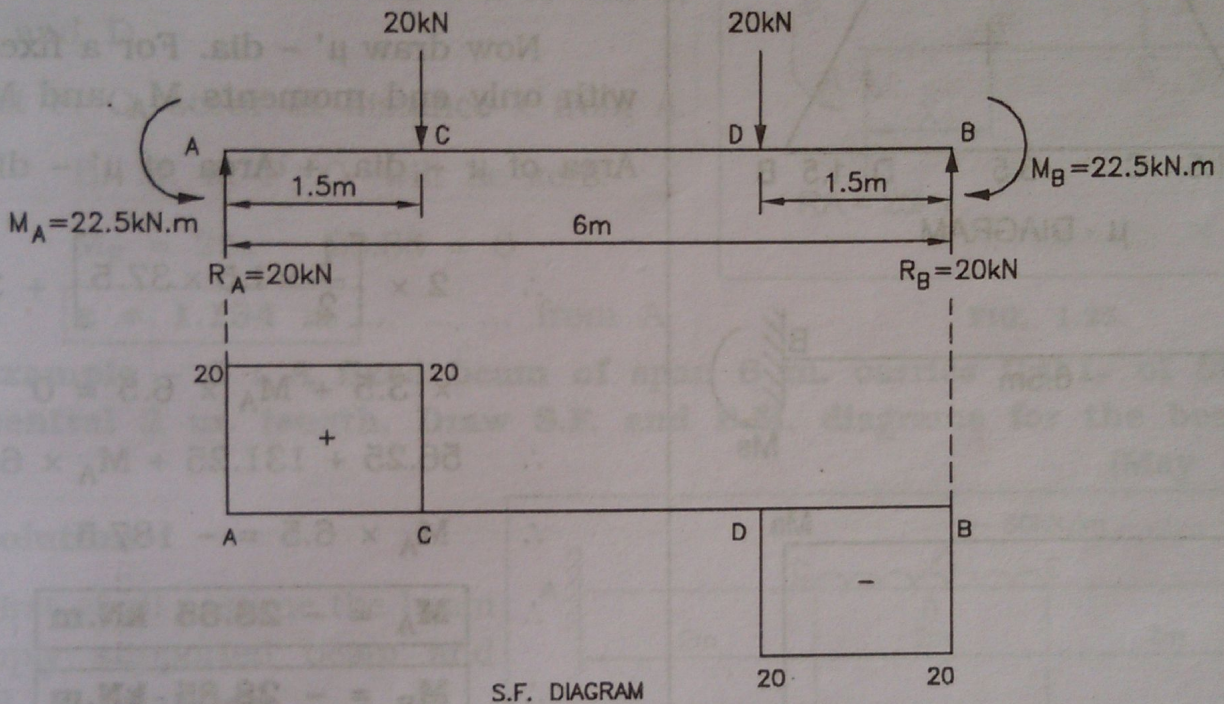


FIG. 1.21

Point of Contraflexure :

The P. C. will occur between A and C, and D and B.

Let P. C. occur at distance x from A.

\therefore B. M. at P. C. will be zero.

$$\therefore M_F = 20x - 22.5 = 0$$

$$\therefore 20x = 22.5$$

$$\therefore \boxed{x = 1.125 \text{ m}} \dots \dots \dots \text{ from A}$$

